Seismic Exploration And Probability:

A Unique Approach to Site Parametric Characterization

In Partial
Fulfillment For The
Requirements Of
Geos 586

Submitted by: John A. Mundell
Submitted to: Prof. Prescott
Date submitted: March 1980

Introduction

In the last several decades, the use of geophysical exploration methods with regard to helping solve geotechnical engineering problems has greatly increased as field methods have become better established and methods of analysis and interpretation have grown in sophistication and accuracy. Because of the increasing costs in site investigations and preliminary studies, the drilling of bore holes for sampling purposes is being challenged by many other potentially feasible methods of site characterization, one of which is geophysical exploration.

Soil profiles are often erratic and soil properties highly variable. In the past the geotechnical engineer has had to rely on a few select test results that would be used to establish the "soil properties" of the site. These tests were run on only a few small samples taken from isolated locations at selected points in time.

With the increased usage of probabilistic methods in soils engineering, as well as geophysical methods which have the capability of characterizing large volumes of the soil mass at a site, instead of small selected volumes, the possibility arises in somehow combining these two areas for the purpose of providing the engineer with insights into the variability of not only the soil stratigraphy and inhomogeneities, but also the distribution of actual soil parameters in the area of interest. With these insights, the engineer may then be able, using sound reasoning and good judgement, to predict the soil performance as well as a given structure's performance.

Because of the multitude of geophysical methods now in use and in the process of further development (the Army Corp of Engineers lists 41), it would be a difficult task to discuss in detail each and every method and its possible application to geotechnical engineering. Therefore, the area of seismic exploration methods including refraction, uphole/downhole, and crosshole surveying will be discussed in detail. These methods are both well developed and potentially adaptable to the probabilistic approaches warranted in certain types of geotechnical problems.

This paper will be divided into three general areas: seimic methods of surface and subsurface exploration including the refraction, uphole/downhole, and crosshole methods; some direct applications of these methods to specific engineering problems; a probabilistic approach using the results of these methods to determine distributions of specific soil parameters to be used in analysis and design.

It is felt that concentration on the three specific seismic methods and their present applications, as well as the development of a procedure of combining these methods with probabilistic ideas, will provide the basis for further insights into the many possible ways that probability and geophysical exploration may be used together in the future to enhance our understanding of site variability. With the continued increase in the use of computers, the potential for further development in this same area cannot be overlooked.

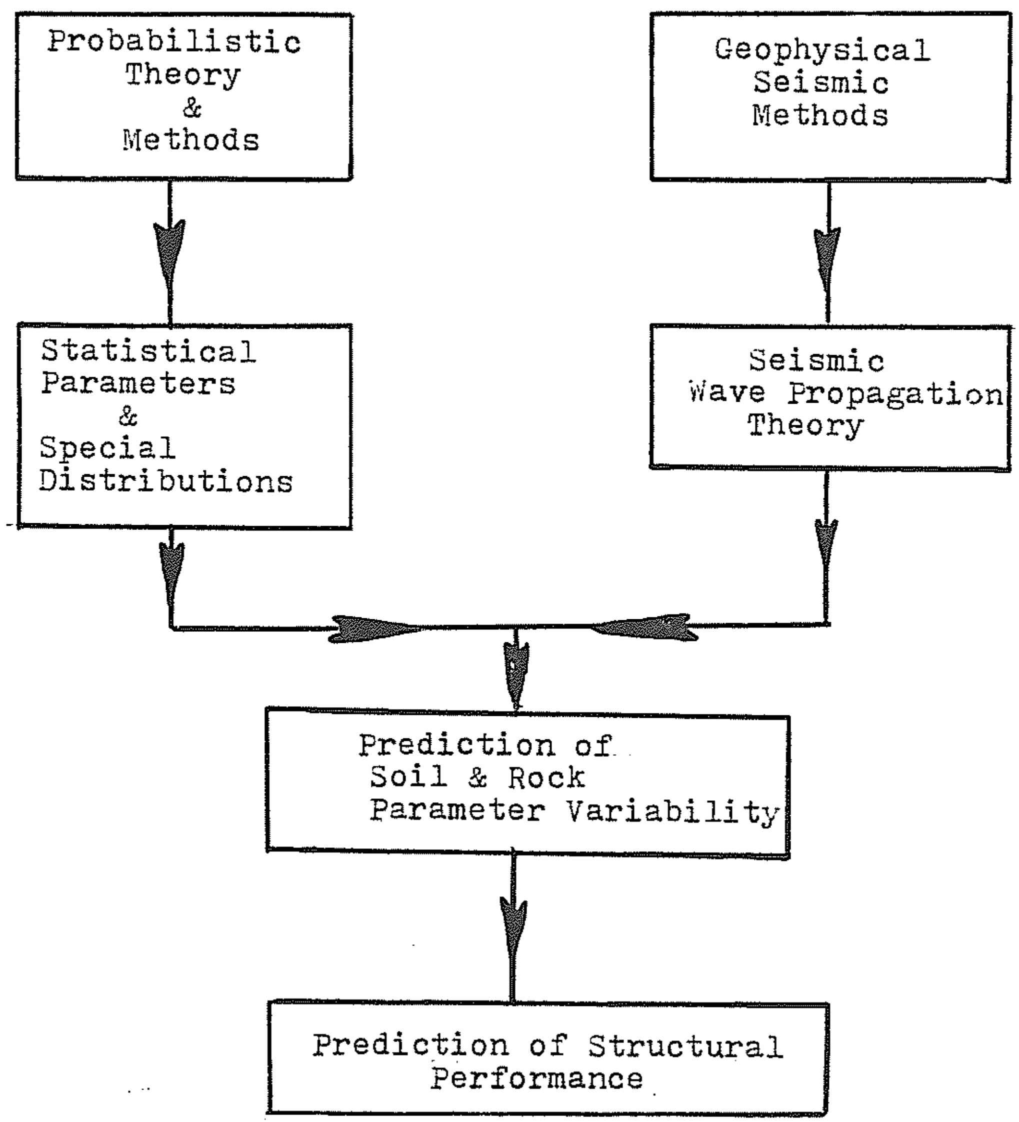


Figure 1. Combining Geophysical and Probalistic Methods for Prediction of Site Variability and Structural Performance.

Seismic Exploration: A Brief Overview

In seismic methods, an elastic pulse or disturbance is created on or below the ground surface, and the resulting motion of the ground at other locations is recorded by small seismometers or "geophones". Measurements of the time interval between the generation of the pulse and its detection at the various geophone locations is an indication of the velocity that the impulse wave propagates through the soil or rock medium.

Since seismic theory is based on the fact that the elastic properties of the subsurface profile directly influence the velocity of propagation, as soils and rock change laterally and with depth, so too should the velocity of propagation, as recorded on different arrangements of geophones. Therefore, any subsurface variability, if distinct enough, should be able to be located and accessed by noting the velocity changes observed in the seismic record.

In the three seismic methods to be discussed, all are based on the "signal source - medium of wave propagation - signal receiver - travel time recorder" concept. Each one approaches the concept with a unique method by which to achieve the desired results. These results are then used to describe some aspect of the subsurface: geometry, lithology, property characteristics. However, before these methods are discussed, it is important to gain a basic understanding of a few of the fundamental ideas that will be used in the coming pages.

Wave Types

Several types of waves may be propagated through an elastic medium, three of which are of interest in our discussion of the seismic methods. One type is the compressional, or P-wave, in which particle motion is parallel to the direction of wave propagation. The P-wave has a greater velocity than the other type of waves and, therefore, is the first to arrive at the geophone detectors. Another wave type is the shear or S-wave, in which particle motion is normal to the direction of wave travel. This wave arrives later at the geophone and its arrival time is often more difficult to interpret than the P-wave. The third wave type that seismic methods may be concerned with is the surface wave (Rayleigh type) in which particle motion is elliptical, involving both shear and compressional movement. Its velocity is only slightly smaller than the S-wave and often interfers with the S-wave's detection on a seismic record. These three wave types are illustrated in figure 2.[10]

Signal Receivers: Geophones

The geophones most often used in seismic surveys for engineering purposes respond to detected impulses by giving an electrical output which is proportional to the vertical or horizontal component of the ground velocity felt at their locations. A common directional type, shown in figure 3, has

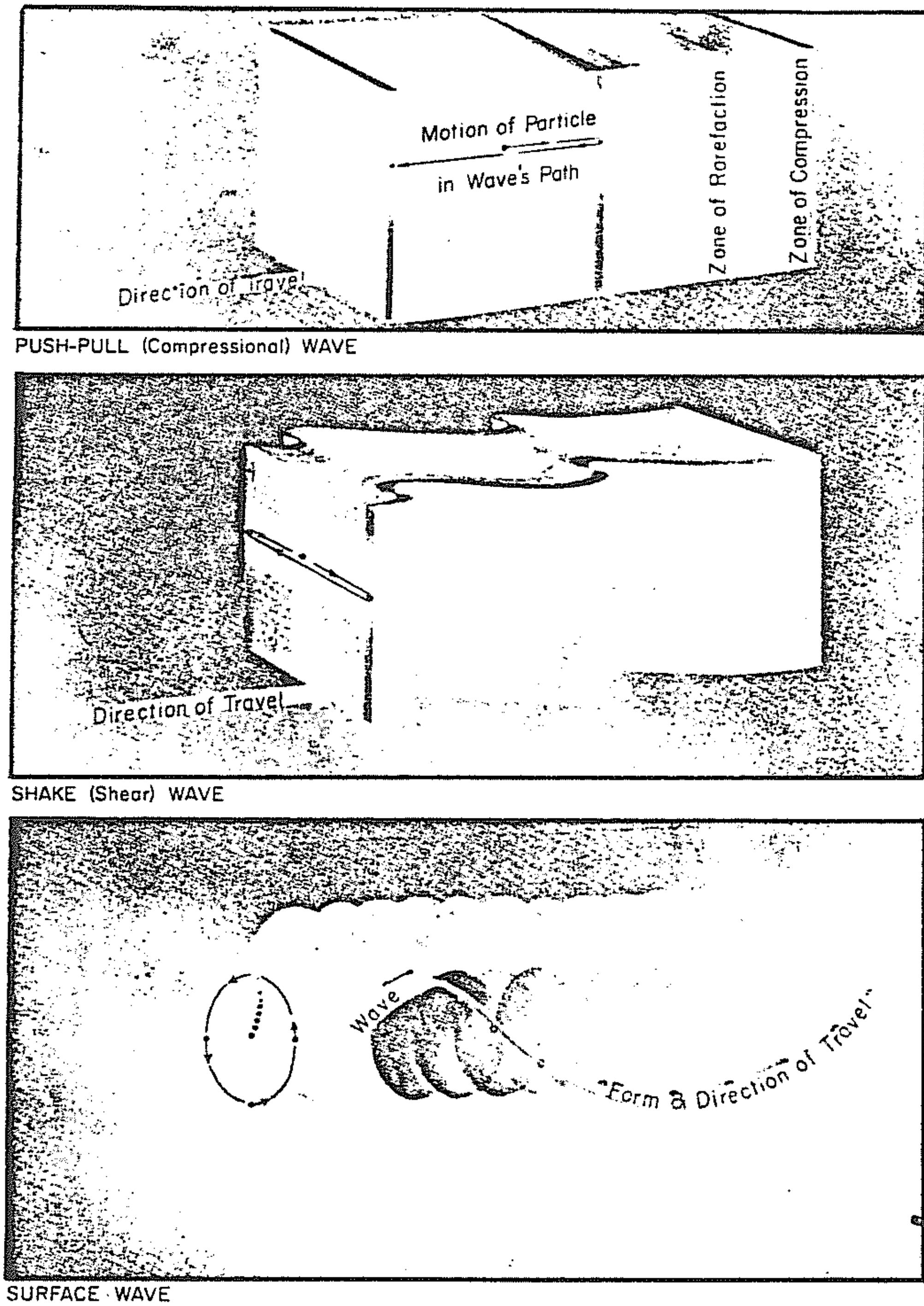
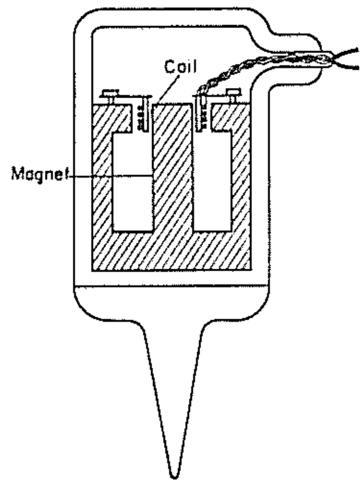


Figure 2. Nature of earth particle displacements during the passage of compressional (P), shear (S), and Rayleigh waves (IN EM 1110-1-1802)

a body which is a permanent magnet. In the radial magnetic field of the gap is a coil on a spring mounting which permits it to move vertically. Any ground motion will cause the coil to move in the field and an electromotive force proportional to the velocity of motion will be induced in the coil.



(Griffiths and King, 1965)

FIG. 3 *Simplified sectional view of a typical moving-coil geophone.*

Depending on the type of wave to be recorded, the directional geophone must be oriented in such a way as to be able to measure the appropriate vertical or horizontal movements caused by the type of wave measured. This will be seen clearly in the geophone orientations used in the seismic methods to be discussed.

Refraction Seismic Survey

The purpose of the refraction survey is to produce a travel time - impact distance graph from the recording of arrival times of the first waves detected at geophones placed a given distance along a line from the wave creation source. A simplified set up of the survey is shown in figure 4.

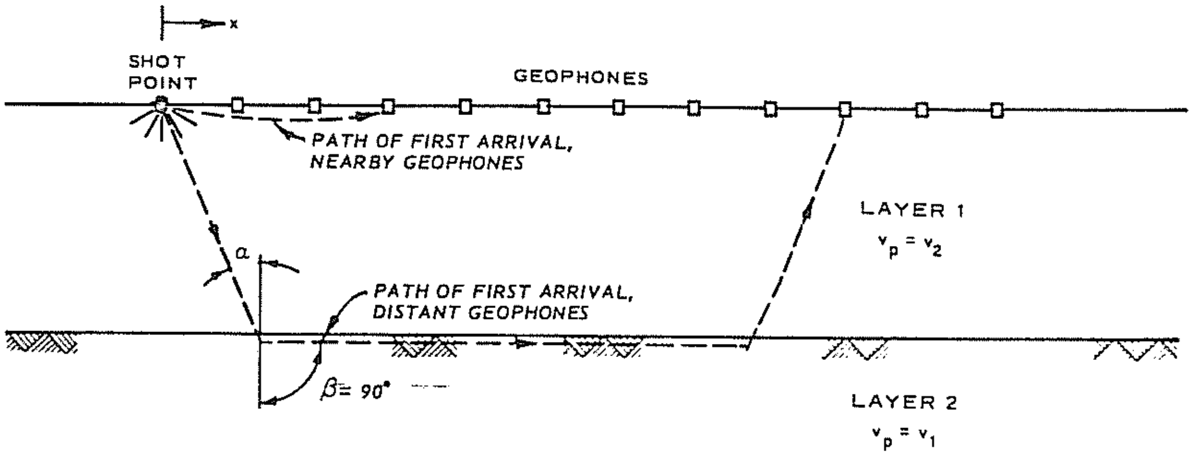


Figure 4. Schematic of seismic refraction survey (prepared by WES)

A hammer blow or small explosive charge is used to induce an elastic wave into the ground. The travel time is the time interval from when the wave is first initiated to the time it is recorded as a disturbance on receiver geophones placed at varying distances from the source. If the travel path of the wave encounters significantly higher velocity layers at different depths, they will produce changes in slopes of the travel-time-impact distance graph that may be interpretted to give the thicknesses and P-wave velocities of the soil,rock layers. Figure 5 shows a typical graph for a multiple layer case.

Problems arise in complex geological subsurface conditions when the time-distance graph is not easily interpretted. The refraction method is not able to determine the existence of low-velocity layers below higher velocity layers, and a "blind zone" or undetected thin layer of stratum is also a

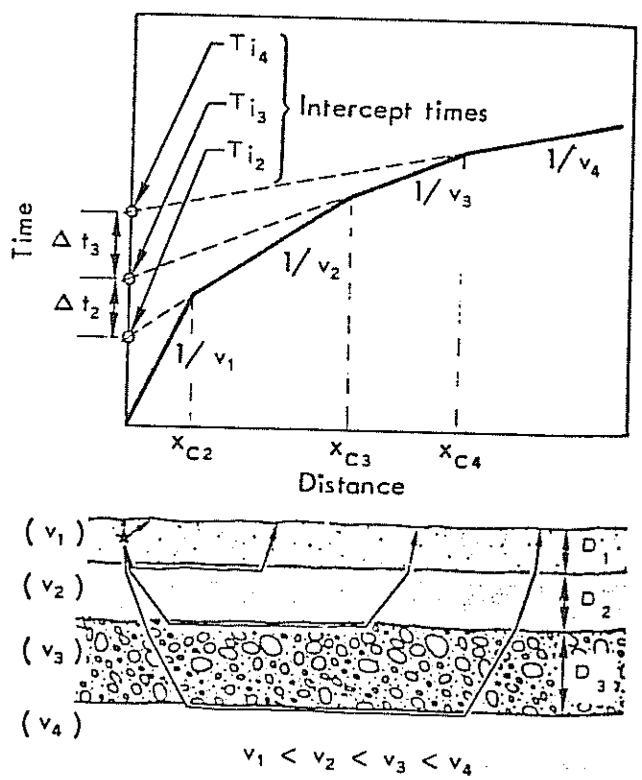


Figure 5. Schematic of multiple-layer case and corresponding time-distance curve (IN EM 1110-1-1802)[10]

common characterisic that the refraction method does not pick up. These complications may give rise to inaccuracies in soil and rock layer thicknesses but, in general, do not affect the P-wave velocities of the different layers calculated from the inverse slopes of the graph.

Subsurface Surveys: Uphole/Downhole/Crosshole Methods

These surveys are made by drilling bore holes and then placing either a seismic energy source or an energy source detector into the hole and measuring the time needed for a seismic wave to travel from source to receiver along a minimum path. Both P-wave and S-wave velocities may be determined through various soil and rock layers at different depths using propagation time and ray path distance-traveled information.

The uphole and downhole methods are illustrated in figures 6, 7, and 8. In the uphole technique, a seismic wave is initiated at the bottom of the hole by some source and the resulting surface disturbance is recorded by geophones located near the mouth of the borehole (usually within 10'). The seismic source is then moved up the hole at specified intervals and the process is repeated until travel times verses depth information has been gathered for the entire borehole.

There are two procedures used in the downhole method, one for obtaining P-wave data, and one for obtaining S-wave data. In both, the seismic recorder is located at a specified depth in the borehole and it records the type of

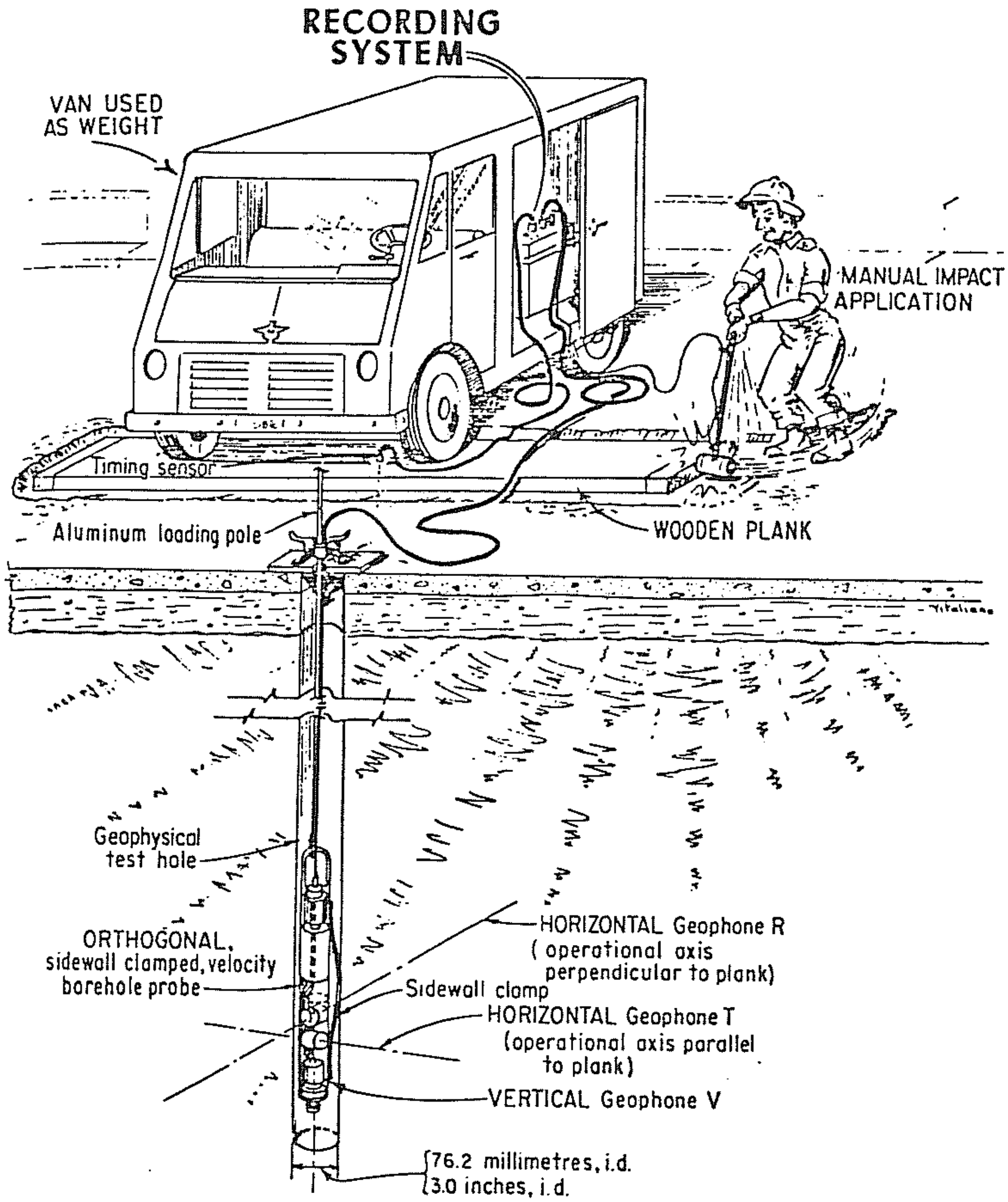


Figure 6. Downhole survey for S-wave data (from Viksne)

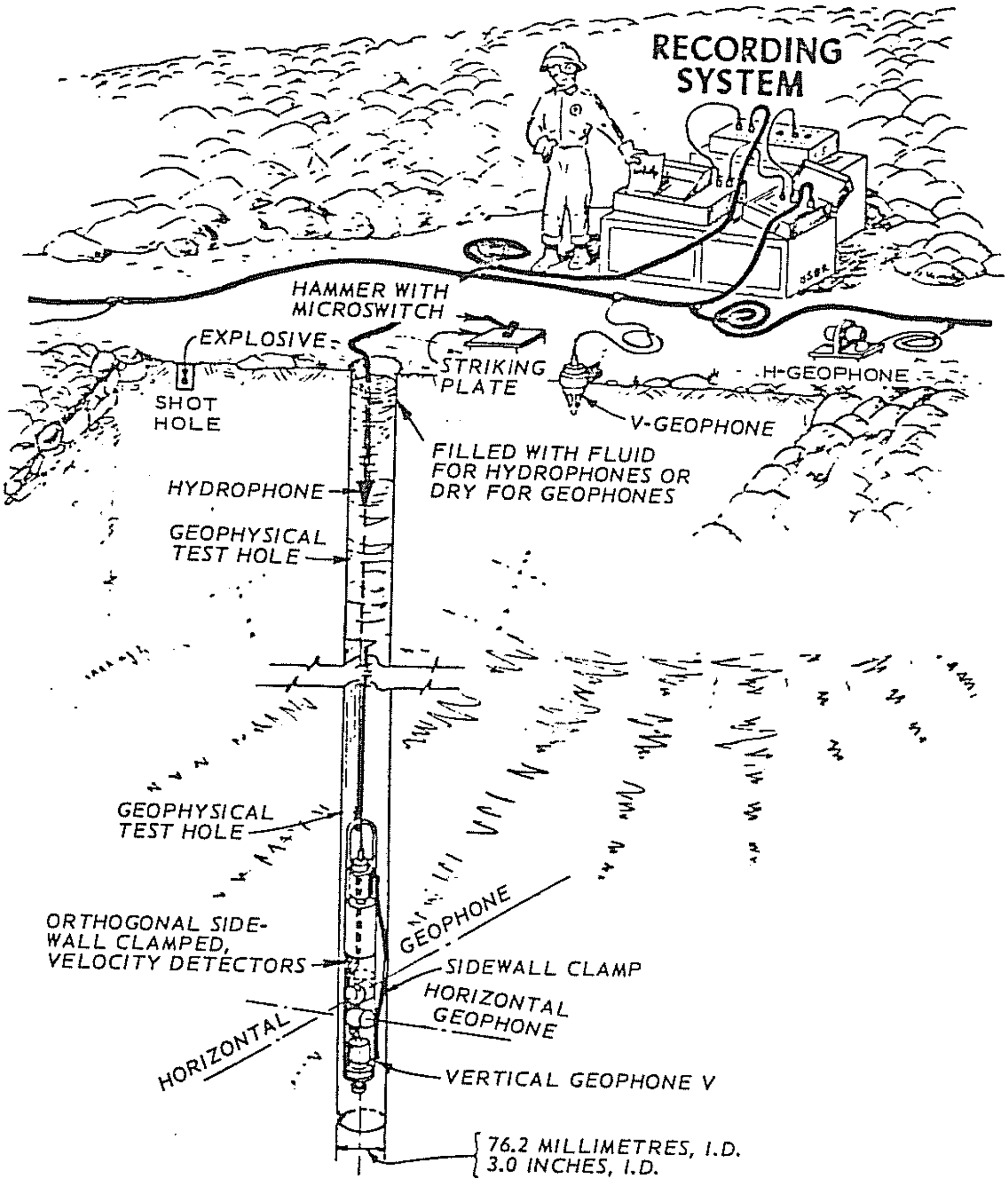


Figure 7. Downhole survey techniques for P-wave data (after Viksne)

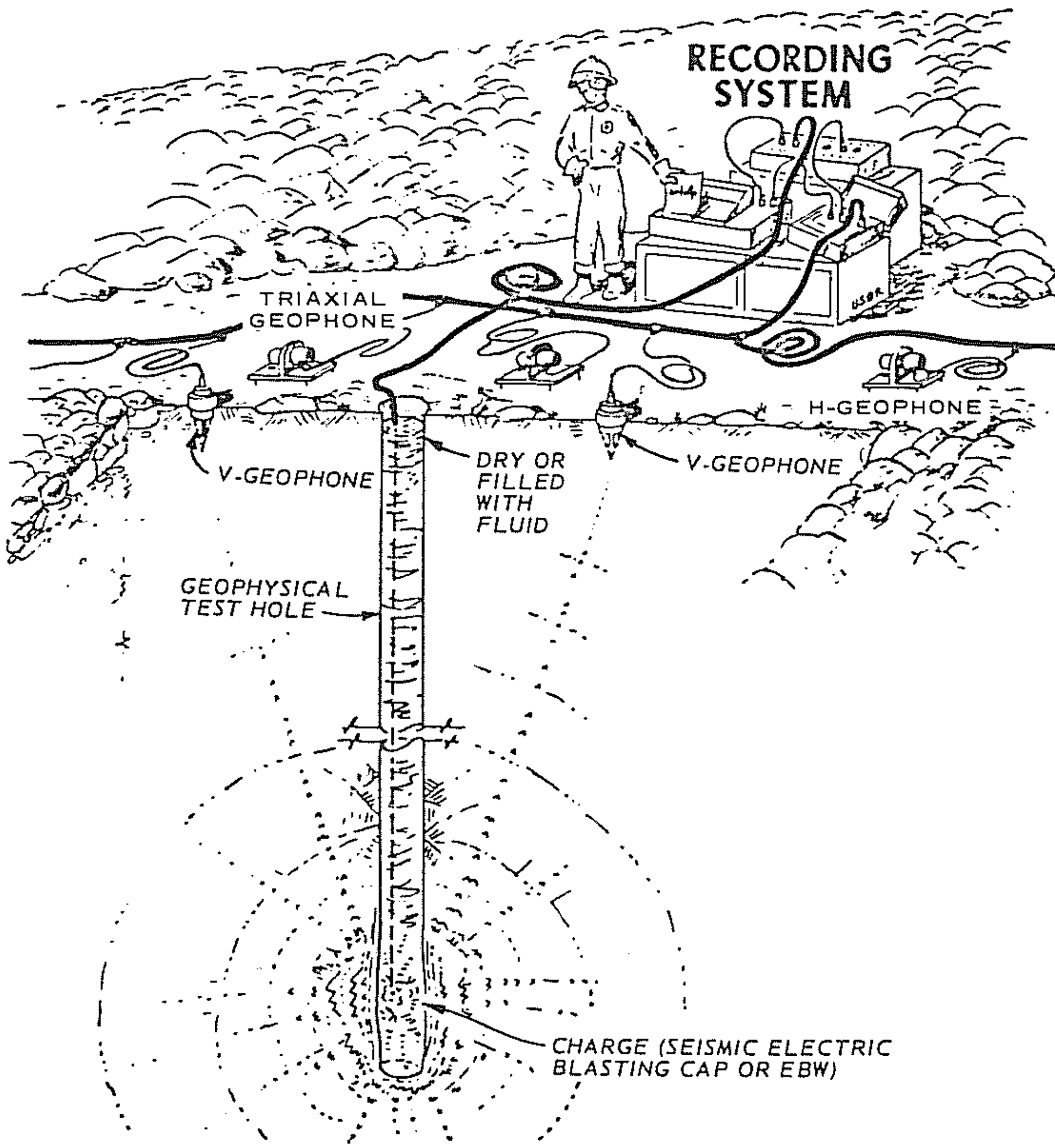
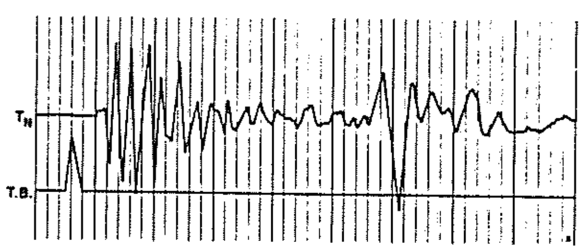
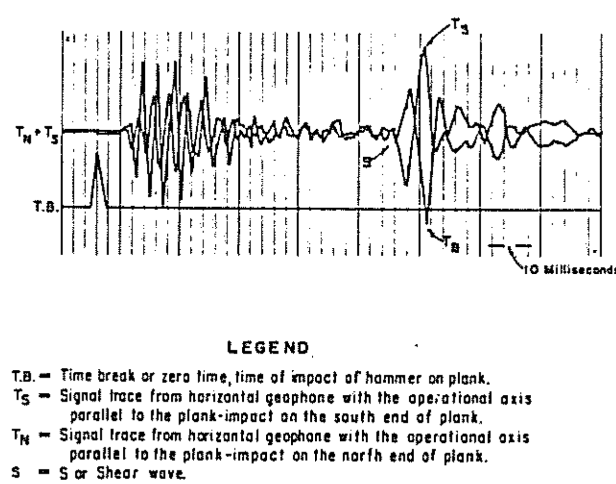


Figure 8. Uphole survey technique
(after Viksne)

wave propagated as well as the time of wave detection. The recorder may be a hydrophone, in which case the drill hole is filled with fluid. The hydrophone records movements in all directions and can thus be used for both P-wave and S-wave detection. The recorder may also be a vertical geophone attached to the side of the bore hole which measures P-wave arrivals, or a triaxial geophone array, which records both P-wave and S-wave arrivals.

If S-wave velocities are desired in a downhole survey, a seismic source consisting of a hammer impacting on the end of a large wooden plank is used. The plank is weighted down so as to provide good contact with the ground surface. In order to identify the S-wave arrival times, the plank is struck on one end, then on the other to provide a 180 phase reversal of the initial shear wave signal. As shown in figure 9, the shear wave arrival may then be able to be more accurately identified.





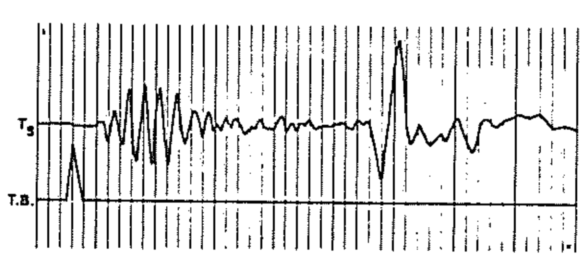


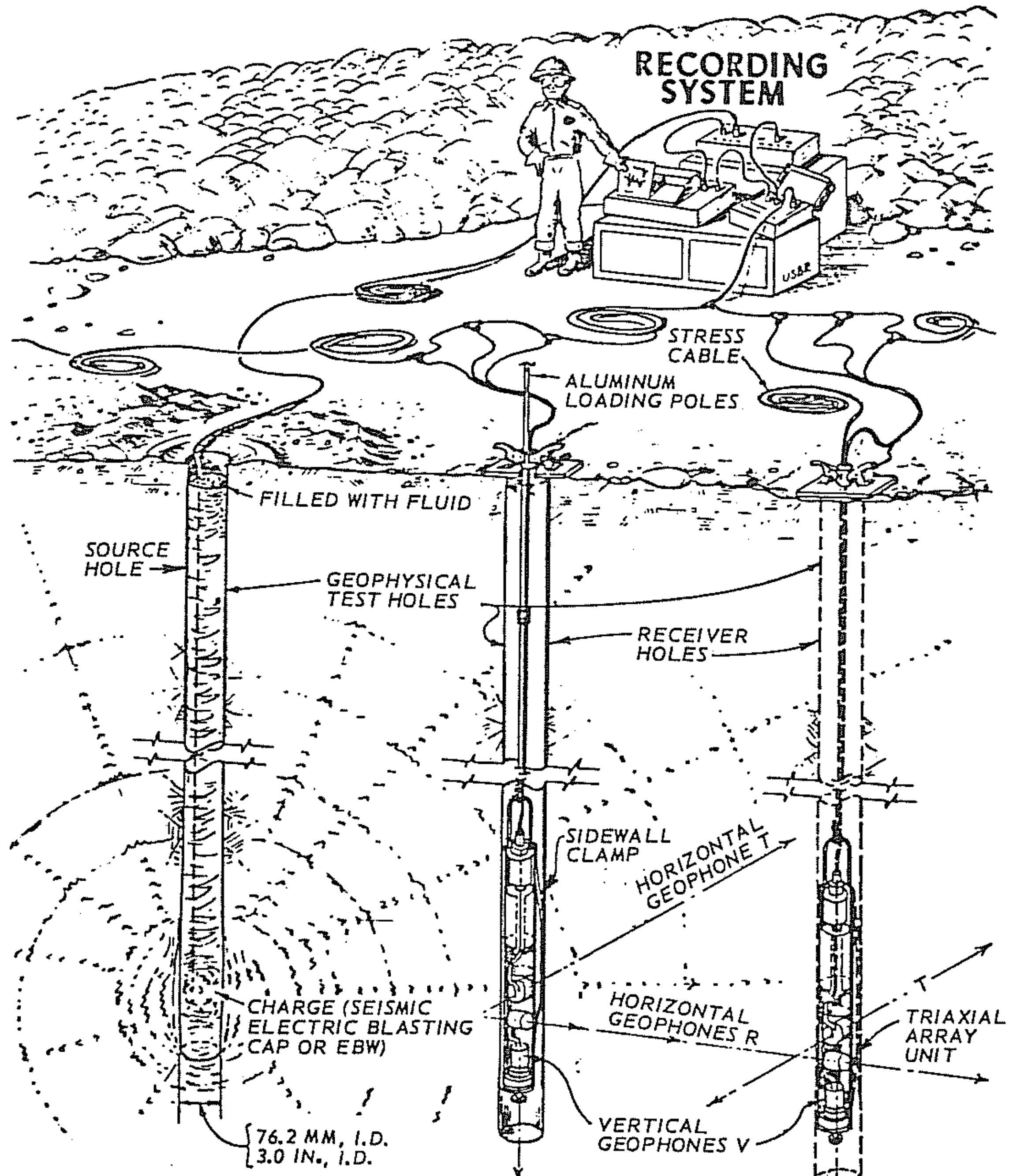
Figure 9. Downhole Phase Reversal

(VIKSNE, 1979)

Crosshole surveys require the need of at least two borings for locations of the seismic source and the receiver set up. As illustrated in figure 10, source and receiver are placed at the same elevation in two bore holes and P-wave and S-wave travel times are measured from the time of the signal initiation to the time of the signal detection by the horizontal and vertical geophones in the other borehole. This is generally done for 5' to 10' intervals in the borehole.

Computer programs, based on Snell's Law of refraction, are used to determine true velocities. They account for zones of high velocity contrasts through which waves may have been refracted to travel to the source in a shorter time. Reliable P-wave velocities may be obtained with calculated values within 5 to 10 per cent of true values. S-wave velocity values are not as easily interpretted because of source problems and difficulty in determining the S-wave arrival times. These velocity values are in the range of 10 to 15 per cent off true values.[10]

The crosshole method gives horizontal P and S-wave velocities as compared to the vertical velocities measured by the uphole and downhole techniques. Variability in the resulting velocities many times is due to vertical and lateral inhomogeneities as well as soil profile anisotrophy. Instead of accounting for this observed variability in the velocities, most often mean values are used in characterizing



NOTE: HYDROPHONES CAN REPLACE GEOPHONES FOR P-WAVE DATA ACQUISITION, BUT RECEIVER HOLES MUST BE FILLED WITH FLUID.

Figure 10. Crosshole test technique for obtaining P- and S-wave data from explosives (after Viksne)

the layers. The probabilistic method to be presented will allow for these variations in values and incorporate them into the design process.

Applications of the Seismic Methods

In general, the refraction, uphole/downhole, and crosshole survey methods previously discussed have been used, to some degree and in combination with other exploration methods, to aid in or accomplish the following:

1. Location, mapping, and correlation of geological features such as stratigraphy, lithology, and their complexities: soil and rock thicknesses and orientations, depth to groundwater, etc.

2. Detection and delineation of buried localized zones of anomalous characteristics: cavities, sink holes, large boulders, zones of weathering and weakness, inclusions of poorly consolidated soils within competent materials, collapsed mine drifts.[5]

3. Preliminary site investigations to provide a basis for: planning subsequent detailed seismic surveys in areas of interest, and locating drill holes in representative areas for the exploration phase of a project.

4. Rippability estimation for excavated material classification: soils and rock with high P-wave velocities are classified as nonrippable materials.

5. Site liquefaction potential studies

6. Elastic settlement predictions using in situ dynamic moduli estimated from P-wave, S-wave, and density field measurements.

7. Measurement of in situ parameters, especially the elastic moduli of soils and rock, in cases where the soil-structure interaction is a dominant factor in the response of the structure to dynamic loading conditions. Examples are: problems of blast loading conditions; wave loading of offshore structures; concrete and earthen dams subject to earthquake loadings; vibratory loading conditions on foundations induced by machinery; foundations for reactor containment vessels, antennas, and other displacement or acceleration sensitive equipment where in situ elastic moduli are sometimes required for design or safety analyses.[10]

It is this last application, that of obtaining a measurement of in situ soil parameters, specifically the dynamic moduli, that will be concentrated on in the probabilistic approach undertaken in the last section of this paper. It is because of the increased acceptance of in situ parameter estimations obtained by geophysical seismic exploration used in the design and/or analysis of a specific structure, that there is a need for the quantification of the variability involved in the derived parameters. If soil parameters obtained through seismic survey data are to be used in design and prediction of structural performance, then the nature of the variability of those parameters must be determined as well as the upper and lower bounds of the parameter distribution. A more detailed discussion of this is included in the last section.

Dynamic Analysis of Dams: A Case History of Houser Lake Dam, Montana 5

In order to make a case for the use of a probabilistic approach to predict the variability of the dynamic soil moduli obtained from seismic methods, an actual case history will be discussed here in which the dynamic soil moduli determined from crosshole surveys across an old concrete dam structure were used to access the stability of the structure.

The problem of doing a dynamic analysis of the foundation of Houser Lake Dam in Montana involved the use of seismic crosshole survey measurements taken along the crest and on the downstream face of the dam. P-wave and S-wave velocities were obtained and the elastic moduli computed by drilling boreholes 60 to 80 feet through the concrete structure and into the bedrock foundation. Seismic waves were then propagated through the dam in the crosshole method previouly described.

Of interest was the detection of physical irregularities, defects, and possible voids in the concrete that would affect the elastic moduli of the concrete. Defects in the concrete such as loose aggregate, no matrix, and the occurrance of fracturing seemed to correlate well with zones of low P-wave velocities. Physical defects ranging from 3 to 10 feet in dimension were able to be detected. The elastic moduli in different areas and depths of the dam computed from the crosshole seismic survey velocity data were used to establish the extent of the defects and, finally, determine the integrity of the concrete dam structure and its foundation.

The use of the crosshole method in this case study to predict the performance of the Houser Lake Dam by estimating dynamic in situ elastic moduli points strongly to the potential use of geophysical techniques as viable means of soil parameter estimation. If combined with probabilistic concepts, new and useful ways of gathering information about site parametric variability and predicting structural integrity and performance may be developed to make site investigation and characteriztion both more reliable and more economical.

Combining Geophysical Exploration and Probabilistic Concepts

Because geophysical methods are able to "evaluate" large volumes of soil and rock over vast areas economically and efficiently, and, variations in the resulting measurements due to soil property variations can be observed and quantified, the application of probabilistic techniques to the information gathered in a geophysical survey can provide insights that may be incorporated with a great deal of confidence, into actual foundation design and evaluation procedures.

In this section, the variability of the in siu dynamic moduli will be examined.. These moduli can be calculated from relationships with known values of P-wave and S-wave velocities and soil densities.

Elastic Properties of a Material

The theory of elasticity states that in the interior of an elastic, isotropic, homogeneous body, two kinds of seismic waves may be propagated, the P-wave and the S-wave. The speed, or seismic velocity, of these waves depends a great deal on the elastic properties of the material. To describe the elasticity of a material completely, two of the following elastic constants need to be specified:

1. Bulk Modulus (incompressibility) - B
2. Shear Modulus - G
3. Young's Modulus - E
4. Poisson's Ratio - v

The theoretical relationships existing between the elastic constants and the seismic velocities are shown below:

$$v = \left[(V_P/V_S)^2 - 2\right] / \left[2(V_P/V_S)^2 - 2\right]$$

$$E = 2(1+v)V_S^2\rho$$

$$B = \rho V_P^2 - \tfrac{4}{3}\rho V_S^2$$

$$G = \rho V_S^2$$

where: V_P = P-wave velocity
V_S = S-wave velocity
ρ = density

The objective of the approach to be used is to determine the overall variability of each of the dynamic elastic moduli by accounting for the individual variabilities of the P-wave and S-wave velocities and the densities. Once the variabilities of the elastic constants are determined, the distribution or "probabilities of occurrence" for the constants will be estimated. This information can then be used in the prediction of the performance of given structures if design limits are placed on the maximum and minimum allowable elastic constant values used in design calculations.

Taylor's Series 1st Order Approximation Method [3]

The method described here will be used to estimate the variabilities of the elastic constants knowing the variabilities of each of the variables in the elastic equations on the last page.

The Taylor's series approximation method enables the mean and variance of the dependent variable y in the function $y = F(x_1, x_2, x_3, \ldots, x_N)$ to be determined given the mean values μ_i and variance S_i^2 of each of the independent random variables $x_1, x_2, x_3, \ldots, x_N$.

Mean Value, y

$$\bar{y} = F(\mu_1, \mu_2, \mu_3, \ldots, \mu_N)$$

Variance, V y

$$V[y] = \sum_{i=1}^{N} \left(\frac{\partial F}{\partial x_i}\right)^2 (S_i^2)$$

All derivatives are evaluated at their respective mean values. For example, if:

$$y = F(x_1, x_2, x_3) = x_1^2 x_2^3 x_3^4$$

$$\frac{\partial F}{\partial x_1} = (2x_1) x_2^3 x_3^4$$

$$\frac{\partial F}{\partial x_2} = x_1^2 (3x_2^2) x_3^4$$

$$\frac{\partial F}{\partial x_3} = x_1^2 x_2^3 (4x_3^3)$$

Then:

$$\bar{y} = \bar{x}_1^2 \bar{x}_2^3 \bar{x}_3^4$$

$$V[y] = \left[(2x_1) x_2^3 x_3^4\right] S_{x_1}^2 + \left[x_1^2 (3x_2^2) x_3^4\right] S_{x_2}^2 + \left[x_1^2 x_2^3 (4x_3^3)\right] S_{x_3}^2$$

The rest of the statistical parameters for the dependent variable y may then be determined:

<u>Standard Deviation, S_y</u>

$$S_y = \sqrt{V[y]}$$

<u>Coefficient of Variation, V_y</u>

$$V_y = \frac{S_y}{\bar{y}}$$

This procedure was used for the deterministic equations involving the elastic constants and the P-wave and S-wave velocities and the densities. The resulting relationships are shown in figure 11.

A computer program was written using these relationships and input data of the P-wave and S-wave velocities and the soil densities, and their respective coefficients of variation. The capability of the program is to calculate the mean, variance, Mean Value Relationships

$$\bar{v} = \left[(V_p/V_s)^2 - 2\right] / \left[2(V_p/V_s)^2 - 2\right]$$
$$\bar{E} = 2(1 + v)V_s^2 \rho$$
$$\bar{B} = \rho V_p^2 - \frac{4}{3}\rho V_s^2$$
$$\bar{G} = \rho V_s^2$$

Variance Relationships

$$V[v] = \left[(V_p/V_s^2)/((V_p/V_s)^4 - 2(V_p/V_s)^2 + 1)\right]^2 (S_{V_p}^2) \; + \ldots$$
$$\left[(-V_p^2/V_s^3)/((V_p/V_s)^4 - 2(V_p/V_s)^2 + 1)\right]^2 (S_{V_s}^2)$$
$$V[E] = \left[2(1 + v)V_s^2\right]^2 (S_{V_p}^2) + \left[2\rho V_s^2\right]^2 (S_v^2) + \left[4(1 + v)\rho V_s^2\right]^2 (S_{V_s}^2)$$
$$V[B] = (V_p^2 - \frac{4}{3}V_s^2)^2 (S_\rho^2) + (\frac{-8}{3}\rho V_s)^2 (S_{V_s}^2) + (2\rho V_p^2)^2 (S_{V_p}^2)$$
$$V[G] = (V_s^2)^2 (S_\rho^2) + (2\rho V_s)^2 (S_{V_s}^2)$$

Note: Use mean values of all independent variables in the above expressions.

Figure 11. Mean value and variance relationships for the Dynamic Moduli.

standard deviation, and coefficient of variation of the dynamic elastic moduli given the input data and the derived relationships. Using the computer program, a study on the effects of changing input parameter variability on the elastic constants was carrited out.

Study on the Effects of Increasing Variability on Elastic Constants

For the study of elastic constant variability due to input data variability, the following data sets were used:

1. V_p = 1000 meters/second $\quad \frac{V_s}{V_p} = 0.2$
   V_s = 200 meters/second
   V_{V_p} = 5, 10, 15, 20 %
   V_{V_s} = 5, 10, 15, 20 %
2. V_p = 1000 meters/second $\quad \frac{V_s}{V_p} = 0.4$
   V_s = 400 meters/second
   V_{V_p} = 5, 10, 15, 20 %
   V_{V_s} = 5, 10, 15, 20 %
3. V_p = 1000 meters/second $\quad \frac{V_s}{V_p} = 0.6$
   V_s = 600 meters/second
   V_{V_p} = 5, 10, 15, 20 %
   V_{V_s} = 5, 10, 15, 20 %

All data sets: ρ = 200 kilograms/cubic meter
V_ρ = 5 %

It was assumed that the density of any layer, as determined by other testing procedures (eg. lab tests, logging procedures), would not have a significant degree of variability within that layer and, therefore, was kept constant with a small degree of variation (V_{ρ} = 5%) so as to observe more closely the effects of the seismic velocities on the elastic constants. If, however, the density is suspected of being highly erratic within a given deposit, its variability effect on the elastic constants could be determined in a similar procedure.

Results of the variability study are shown in Table 1, with the coefficients of variation given for all of the dynamic moduli. Figure 12 contains the listing of the computer program written to analyze the input data with a typical output shown. A portion of the total results (for $\frac{V_s}{V_p}$ = 0.2) is included in the Appendix for closer examination. In general, the conclusions that can be drawn from the results are:

<u>Young's Modulus, E</u>

1. The $\frac{V_s}{V_p}$ ratio has a small effect on V_E. As it increases, V_E slightly increases also.
2. V_{V_p} has almost no effect on V_E.
3. As V_{V_s} increases or decreases by a certain magnitude, V_E roughly does the same.

<u>Poisson's Ratio, v</u>

1. The $\frac{V_s}{V_p}$ ratio has a great effect on V_v. As it increases, V_v also increases, but at a much greater rate.
2. For a small $\frac{V_s}{V_p}$ ratio (0.2):

   a.) Changes in V_{V_p} or V_{V_s} have very little effect on V_v.

TABLE 1. Results of Variability Study

V_{V_P} (%)	V_{V_S} (%)	V_E (%)			V_V (%)			V_B (%)			V_G (%)		
		* 0.2	* 0.4	* 0.6	* 0.2	* 0.4	* 0.6	* 0.2	* 0.4	* 0.6	* 0.2	* 0.4	* 0.6
5	5	11.18	11.24	12.29	0.64	3.96	28.41	11.70	13.93	21.91	11.18	11.18	11.18
5	10	20.62	20.69	22.14	1.01	6.26	44.92	11.74	14.70	27.12	20.62	20.62	20.62
5	15	30.42	30.52	32.48	1.43	8.86	63.53	11.81	15.90	34.08	30.41	30.41	30.41
5	20	40.32	40.45	42.97	1.87	11.55	82.83	11.90	17.44	41.93	40.31	40.31	40.31
10	5	11.19	11.33	13.78	1.01	6.26	44.92	21.72	26.05	39.87	11.18	11.18	11.18
10	10	20.62	20.74	23.00	1.28	7.92	56.82	21.74	26.47	42.95	20.62	20.62	20.62
10	15	30.42	30.55	33.08	1.63	10.10	72.43	21.78	27.16	47.66	30.41	30.41	30.41
10	20	40.32	40.47	43.42	2.03	12.53	89.84	21.83	28.09	53.55	40.31	40.31	40.31
15	5	11.19	11.47	15.97	1.43	8.86	63.53	32.09	38.56	58.64	11.18	11.18	11.18
15	10	20.62	20.82	24.37	1.63	10.10	72.43	32.10	38.84	60.78	20.62	20.62	20.62
15	15	30.42	30.61	34.04	1.92	11.88	85.23	32.13	39.31	64.19	30.41	30.41	30.41
15	20	40.32	40.51	44.16	2.26	14.01	100.4	32.16	39.96	68.68	40.31	40.31	40.31
20	5	11.20	11.67	18.60	1.87	11.55	82.83	42.55	51.16	77.64	11.18	11.18	11.18
20	10	20.63	20.93	26.17	2.03	12.53	89.84	42.56	51.38	79.27	20.62	20.62	20.62
20	15	30.42	30.68	35.36	2.26	14.01	100.4	42.58	51.74	81.91	30.41	30.41	30.41
20	15	40.32	40.57	45.18	2.56	15.85	113.6	42.61	52.23	85.47	40.31	40.31	40.31

* $\frac{V_S}{V_P}$ ratio

For V_P = 1000 m/sec
V_S = 200, 400, 600 m/sec
ρ = 200 kg/cubic meter
V_e = 5 %

```
UNIVERSI  OF MINNESOTA FORTRAN COMPILER (VERSION . 5 - 10/27/78) ON PURDUE DUAL 6500 MACE S  TEM  03/16/
                     MNF.
                     C
                     C*********************************************************
                     C                                                        *
                     C   GEOPHSICAL PARAMETER VARIABILITY STUDY               *
                     C                                                        *
                     C       DEVELOPED BY JOHN A MUNDELL MARCH 1980           *
                     C                                                        *
                     C       THIS PROGRAM DETERMINES THE EFFECT OF            *
                     C       GEOPHYSICAL DATA VARIABILITY ON THE DYNAMIC      *
                     C       ELASTIC MODULI USING THE TAYLOR SERIES           *
                     C       METHOD AS OUTLINED IN HARR(1975).                *
                     C                                                        *
                     C   THE INPUT SYMBOLS ARE AS FOLLOWS...                  *
                     C                                                        *
                     C       VP = COMPRESSIONAL WAVE VELOCITY                 *
                     C       COEFVP = COEFFICIENT OF VARIATION OF VP          *
                     C       VS = SHEAR WAVE VELOCITY                         *
                     C       COEFVS = COEFFICIENT OF VARIATION OF VS          *
                     C       P = DENSITY OF SPECIFIC LAYER                    *
                     C       COEFP = COEFFICIENT OF VARIATION OF P            *
                     C                                                        *
                     C*********************************************************
                     C
                     C
   1.     000000B            REAL MEANV,MEANE,MEANB,MEANG,P
                     C
                     C   VELOCITIES ARE INPUT IN METERS/SECOND
                     C   WET DENSITY IS INPUT IN KILOGRAMS/CUBIC METER
                     C
   2.     000000B     1      READ(5,*,END=2) VP,COEFVP,VS,COEFVS,P,COEFP
                     C
                     C   CALCULATE STANDARD DEVIATIONS OF INPUT PARAMETERE
                     C
   3.     002070B            SDVP=(COEFVP/100.)*VP
   4.     002072B            SDVS=(COEFVS/100.)*VS
   5.     002074B            SDP=(COEFP/100.)*P
                     C
                     C   PRINT INPUT DATA
                     C
   6.     002076B            PRINT 99
   7.     002103B            PRINT 100,VP,COEFVP,SDVP,VS,COEFVS,SDVS,P,COEFP,SDP
                     C
                     C   CONVERSION FACTOR TO CONVERT MODULI TO MEGA PASCALS.
   8.     002120B            CONFAC=9.806/10.**6.
                     C
                     C   POISSONS RATIO, V
                     C
   9.     002122B            RATIO=VP/VS
  10.     002123B            VDERVP=(VP/VS**2.)/(RATIO**4.-2.*RATIO**2.+1.)
  11.     002130B            VDERVS=-1.*(VP**2./VS**3.)/(RATIO**4.-2.*RATIO**2.+1.)
                     C
 (11) -   WARNING    ----------------------------------------------------MULTIPLY OR DIVIDE BY 1
  12.     002136B            MEANV=(RATIO**2.-2.)/(2.*RATIO**2.-2.)
  13.     002142B            VARV=(VDERVP**2.*SDVP**2.)+(VDERVS**2.*SDVS**2.)
  14.     002150B            SDV=SQRT(VARV)
  15.     002153B            COEFV=SDV/MEANV
```

Figure 12. Computer Listing

```
                    C
                    C   YOUNGS MODULUS
                    C
        002154B          EDERP=2.*(1.+MEANV)*VS**2.
        002157B          EDERV=2.*P*VS**2.
        002161B          EDERVS=4.*(1.+MEANV)*P*VS
        002163B          EDE2VS=4.*(1.+MEANV)*P
        002166B          MEANE=2.*(1.+MEANV)*P*VS**2.*CONFAC
        002171B          VARE=((EDERP**2.*SDP**2.)+(EDERV**2.*SDV**2.)+(EDERVS**2.*SDVS**2.
                        *))*CONFAC**2.
        002204B          SDE=SQRT(VARE)
        002206B          COEFE=SDE/MEANE
                    C
                    C   BULK MODULUS
                    C
        002207B          BDERP=VP**2.-(4./3.)*VS**2.
        002213B          BDERVS=(-8./3.)*P*VS
        002215B          BDERVP=2.*P*VP
        002216B          BDE2VS=(-8./3.)*P
        002217B          BDE2VP=2.*P
        002220B          MEANB=(P*VP**2.-(4./3.)*P*VS**2.)*CONFAC
        002223B          VARB=((BDERP**2.*SDP**2.)+(BDERVS**2.*SDVS**2.)+(BDERVP**2.*SDVP**
                        *2.))*CONFAC**2.
        002236B          SDB=SQRT(VARB)
        002240B          COEFB=SDB/MEANB
                    C
                    C   SHEAR MODULUS, G
                    C
        002241B          GDERP=VS**2.
        002242B          GDERVS=2.*P*VS
        002244B          GDE2VS=2.*P
        002245B          MEANG=P*VS**2.*CONFAC
        002247B          VARG=((GDERP**2.*SDP**2.)+(GDERVS**2.*SDVS**2.))*CONFAC**2.
        002256B          SDG=SQRT(VARG)
        002260B          COEFG=SDG/MEANG
                    C
        002261B          PRINT 101
        002266B          PRINT 110,MEANE,VARE,COEFE,SDE
        002276B          PRINT 102
        002302B          PRINT 110,MEANV,VARV,COEFV,SDV
        002312B          PRINT 103
        002316B          PRINT 110,MEANB,VARB,COEFB,SDB
        002326B          PRINT 104
        002332B          PRINT 110,MEANG,VARG,COEFG,SDG
                    C
        002342B     99   FORMAT('1',1X,'INPUT PARAMETERS',/)
        002342B     100  FORMAT(/3X,'COMPRESSIONAL WAVE VELOCITY,    VP = ',E10.3,
                        */4X,'COEFFICIENT OF VARIATION, COEFVP = ',E10.3,
                        */12X,'STANDARD DEVIATION, SDVP = ',E10.3,/,
                        */3X,'SHEAR WAVE VELOCITY,              VS = ',E10.3,
                        */4X,'COEFFICIENT OF VARIATION, COEFVS = ',E10.3,
                        */12X,'STANDARD DEVIATION, SDVS = ',E10.3,/,
                        */3X,'DENSITY,                           P = ',E10.3,
                        */5X,'COEFFICIENT OF VARIATION, COEFP = ',E10.3,
                        */13X,'STANDARD DEVIATION, SDP = ',E10.3,////)
        002342B     101  FORMAT(1X,'YOUNGS MODULUS, E')
        002342B     102  FORMAT(1X,'POISSONS RATIO, V')
        002342B     103  FORMAT(1X,'BULK MODULUS, B')
        002342B     104  FORMAT(1X,'SHEAR MODULUS, G')
        002342B     110  FORMAT(/24X,'MEAN = ',E10.3,
```

Figure 12. Computer Listing (continued)

```
                  */20X,'VARIANCE = ',E10.3,
                  */4X,'COFFICIENT OF VARIATION = ',E10.3,
                  */10X,'STANDARD DEVIATION = ',E10.3,//)
              C
      002342B       GOTO 1
      002342B   2   STOP
      002345B       END
      CAUTION   ------------------------------------------GDE2VS WAS SET BUT NEVER USED
      CAUTION   ------------------------------------------EDE2VS WAS SET BUT NEVER USED
      CAUTION   ------------------------------------------BDE2VS WAS SET BUT NEVER USED
      CAUTION   ------------------------------------------BDE2VP WAS SET BUT NEVER USED
```

Figure 12. Computer Listing (continued)

3. For a large $\frac{V_s}{V_p}$ ratio (0.6):

   a.) Increasing V_{V_s} or V_{V_p} by a certain magnitude also increases V_V by the same magnitude (roughly).

Bulk Modulus, B

1. The ratio $\frac{V_s}{V_p}$ has an effect on V_B. As it is increased three-fold the value of V_B is increase two-fold.
2. V_{V_s} has a very small effect on V_B. If it is increased, V_B increases only slightly.
3. As V_{V_p} is increased by a certain magnitude, so too does V_B increase by the roughly the same magnitude.

Shear Modulus, G

1. The ratio $\frac{V_s}{V_p}$ has no effect on V_G.
2. V_{V_p} has no effect on V_G.
3. Increasing V_{V_s} by a certain magnitude also increases V_G by roughly the same magnitude.

The results of the above study are important in that they give a general guideline of how the dynamic soil moduli vary as the compressional and shear wave velocities vary (as measured from seismic methods previouly discussed). Heterogeneous soil profiles can then be characterized not by using just the mean values, but also by taking into account the variations or possible distribution of the actual in situ values. In understanding the foundation response to loads, under certain special types of circumstances, it is necessary to be able to predict the likelihood that design values used will be exceeded. This will give the engineer an idea of how well his/her structure will perform.

Given the mean value and variability data for a specific parameter, there are methods that are able to assign a probability or "likelihood of occurrence" to all of the possible values of the parameter.

Estimating Probabilities Using The Beta Distribution

The beta distribution is an empirical distribution which has the ability to model random variable whose values must always be positive quantities, and whose ranges are of a limited extent. The normal and uniform distributions are special cases of the beta distribution and are, therefore, more limited in their applications. The versatility of the beta distribution makes it applicable in many situations where the normal and uniform distributions cannot be applied.

Because of the nature of the dynamic moduli and their variability (as determined from the Taylor's approximation method) the beta distribution is ideal for modeling their occurrence distribution. The objective of this section is to show how the probability of certain values of the dynamic moduli occurring may be estimated, and how the results can be used in predicting soil variability and, in general, structural performance. For a more detailed explanation of the beta distribution and its theoretical basis, refer to Harr(1977).

Example Problem:

To illustrate the procedure to be used while trying to avoid a detailed explanation of the basis of the procedure, an example will be run through to show how the desired results are obtained.

Given: Mean value of Young's Modulus = $\bar{E}$ = 232.1 MPa

Coeff of variation of Young's Modulus = V_E = 20.6 %

Standard Deviation = $S_E = \bar{E} * V_E$ = 47.85 MPa

Step 1: Compute the estimated maximum and minimum values of E by assuming that all E values lie within three standard deviation's of the mean value (in the normal distribution, 99.7% of all values lie within three standard deviations of the mean).

Therefore: $E_{MIN} = \bar{E} - 3S_E = 232.1 - 3(47.85)$

$E_{MIN} = 88.55$ MPa

$E_{MAX} = \bar{E} + 3S_E = 232.1+3(47.85)$

$E_{MAX} = 375.65$ MPa

Step 2: Call $E_{MIN} = a$

$E_{MAX} = b$ and compute the following quantities:

$\tilde{x} = (\bar{E} - a)/(b - a) = (232.1 - 88.55)/(375.65 - 88.55)$

$\tilde{x} = 0.500$

$\tilde{V} = (S_E /(b - a))^2 = (47.85/(375.65-88.55))^2$

$\tilde{V} = 0.0277778$

Step 3: Determine α and β parameters:

$$\alpha = \frac{\tilde{x}^2}{\tilde{V}}(1 - \tilde{x}) - (1 + \tilde{x}) = \frac{(0.5)^2}{0.027778}(1 - 0.5) - (1 + 0.5)$$

$= 3.00$

$$\beta = \left(\frac{\alpha + 1}{\tilde{x}}\right) - (\alpha + 2) = \left(\frac{3.00+1}{0.500}\right) - (3.00 + 2.00)$$

$\beta = 3.00$

Step 4: In figure 13 (taken from Harr(1977)) are given plots that enable the value x_K to be found such that:

$$P[E < x_K] = K\ \%\ ^*$$

* read " The probability that E will be less than x_K is K % ".

The plots are entered with $\alpha + 2$, $\beta + 2$ values and the value F is picked off the plot.

If $x_K = 5\ \%$ is desired in our case, then:

$$\alpha + 2 = 5.00$$
$$\beta + 2 = 5.00$$ and F = 0.22 from the K= 5% plot.

$$x_K = a + F(b - a) = 88.55 + (0.22)(375.65 - 88.55)$$

$$x_K = 151.7 \text{ MPa}$$

Therefore, for this problem, there is a 5% chance that the value of E in the field will be less than 151.7 MPa. The values of E for any K% may be determined and the distribution presented in the form of a cumulative distribution graph. For this problem, this is done in figure 14. With the use of the computer, these results may be generated quite rapidly and the distribution found for any dynamic moduli with its given coefficient of variation.

Application of Probabilistic Results

The last section's method gave results of the following form:

$$P\left[(\text{dynamic moduli})_{\text{FIELD}} < (\text{dynamic moduli})_{\text{DESIGN}}\right] = K\ \%$$

It can be seen that this information may be used in design and

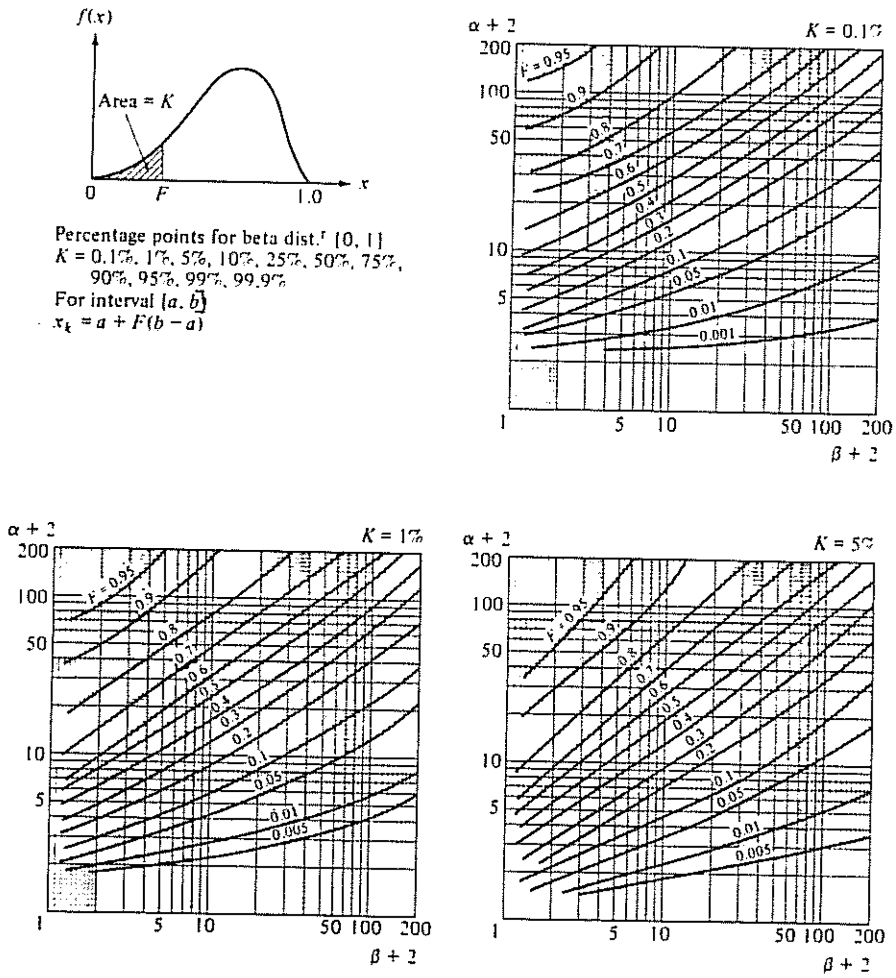


Figure 13 Plots of the percentage points $K = 0.1, 1, 5, 10, 25, 50, 75, 90, 95, 99$ and 99.9 percent for beta distributions 0,1 for the interval $[a, b]: x_k = a + F(b - a)$.

(Harr, 1977)

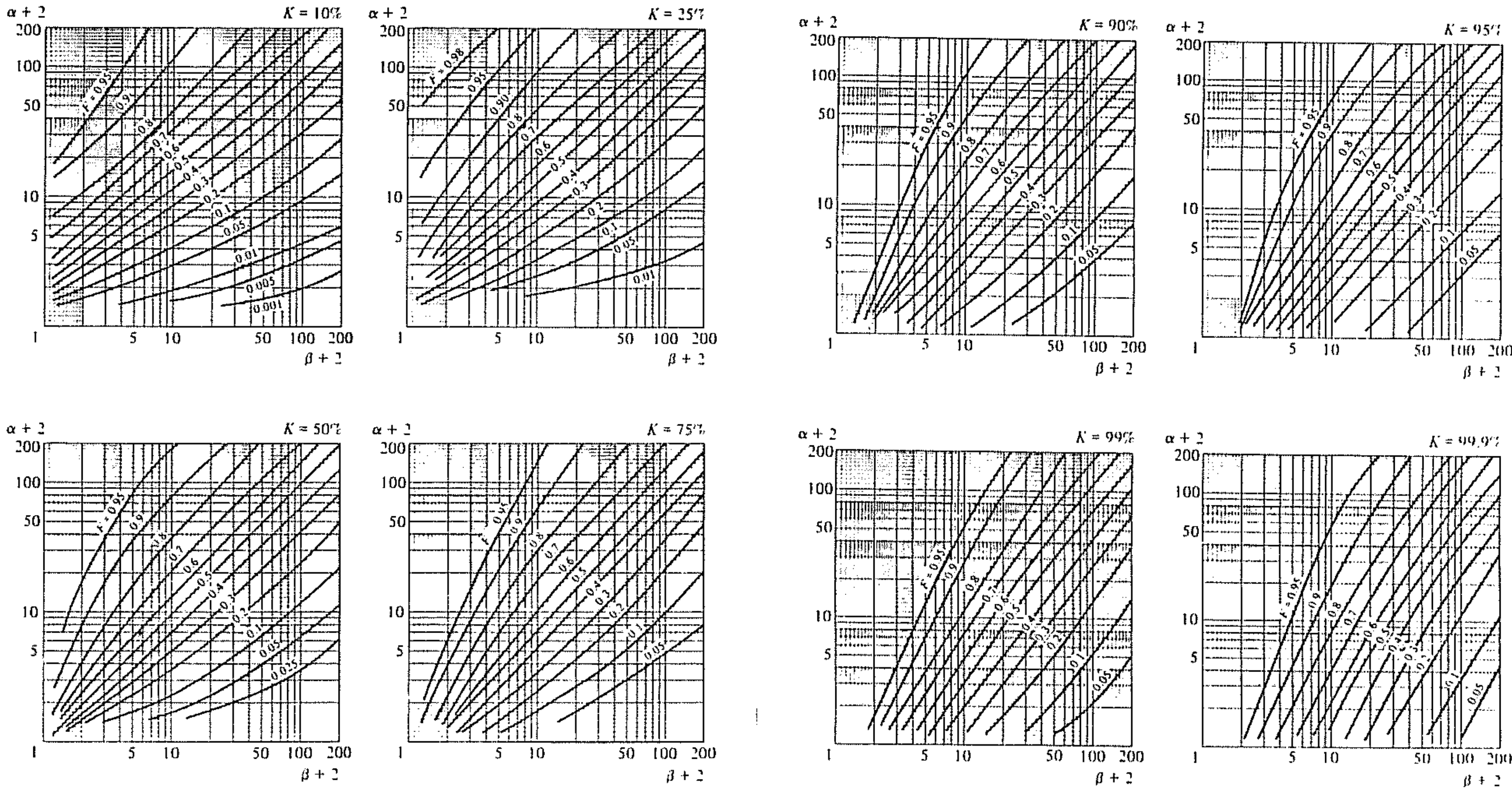
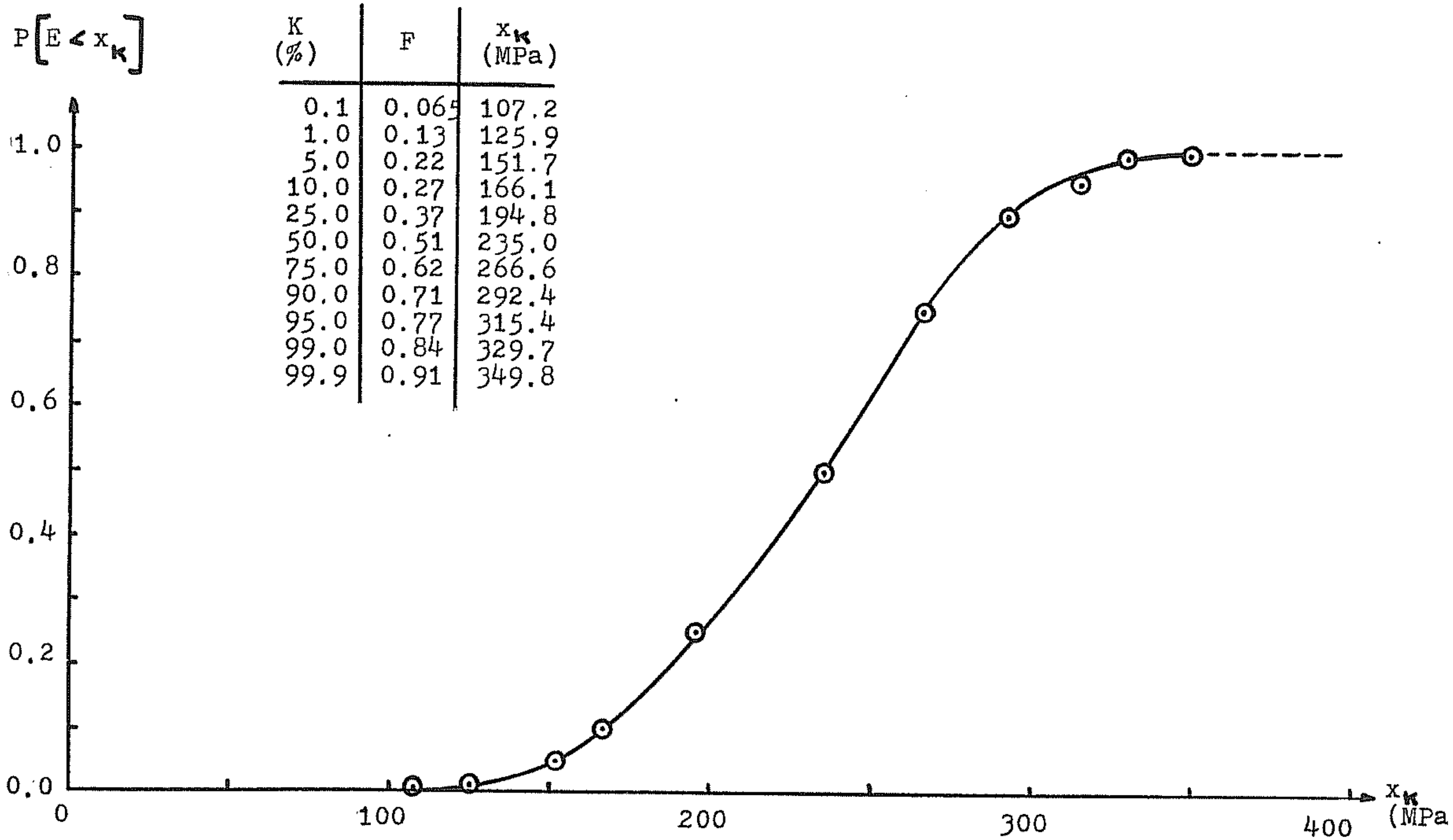


Figure 13 (continued)

(HARR, 1977)



K (%)	F	x_K (MPa)
0.1	0.065	107.2
1.0	0.13	125.9
5.0	0.22	151.7
10.0	0.27	166.1
25.0	0.37	194.8
50.0	0.51	235.0
75.0	0.62	266.6
90.0	0.71	292.4
95.0	0.77	315.4
99.0	0.84	329.7
99.9	0.91	349.8

Figure 14. Cumulative Frequency Distribution Graph for $\bar{E}$ = 232.1 MPa and V_E = 20.6 %.

analysis procedures in which the result directly relies on the value of the dynamic moduli chosen for the situation.

For example, suppose that the failure of a structure or foundation occurs if the in situ value of one of the dynamic moduli is below a certain allowable value, with failure defined as: the inability of the structure to perform its function; an unfavorable soil response (high deformations and stress levels) causing structural distress. Written in another manner:

Probability of Failure = p_f

$$p_f = P\left[(\text{dynamic moduli}) < x_K\right] = K\ \%$$

where x_K is the allowable dynamic moduli used in design.

Therefore, from the probabilistic methods previouly described, variations in the in situ dynamic moduli calculated from variations in the P-wave and S-wave velocities and profile densities may be used to help establish design values to be used that will give the probability of failure "level" desired for specified projects.

<u>Summary</u>

It has been shown that the results obtained from seismic surveys using the refraction, uphole/downhole, and crosshole methods can be used to quantify the variability of the subsurface soil and rock parameters, specifically the in situ dynamic elastic moduli. The magnitude or degree of variation of these moduli depend a great deal on the variability of the P-wave and S-wave velocity and density of each layer, as well as the magnitude of the S-wave velocity/P-wave velocity ratio.

It is believed that as geophysical methods are used more and more in engineering to predict or estimate soil and rock properties and parameters, the use of probabilistic methods in describing and quantifying these parameter's distributions will gain importance in design and analysis procedures for predicting the performance of specific structures.

Although the geophysical methods used in field surveys are, at this time, not as standardized and refined as the lab testing procedures, they still have the advantage of being able to characterize large volumes of material over an entire site. Lab test results, no matter how carefully recorded on even the finest of equipment, are only as good as the sample tested is representative of the actual site conditions. The potential for future development of the present geophysical methods for engineering purposes is still a long ways off from being fully realized, and the increasing use of computer analysis in all areas of engineering adds to the viability of the combination of these two areas for effecient and economical site characterization.

Recommendations

The models developed to predict performance must, more than ever, "encompass uncertainty so as to pinpoint the effects of that uncertainty and its importance to total project performance in a quantatative manner." Qualitative engineering judgement should be aided by quantitative probability-assessment methods.

Establishing the use of probabilistic quantitative aspects in geotechnical exploration must be accompanied by increasing efforts to systematically tabulate the variabilities and distributions of geophysical, geological, and geotechnical data gathered in tremendous quantities presently by many groups: universities, governmental agencies, private business groups. With this information, specific soil and rock types and <u>their</u> natural variability will begin to be established, and thus may be incorporated with increasing confidnece into engineering analysis.

Geophysical methods must continue to be developed and refined for use in the shallow depths associated with most engineering structures. Varification with borehole samples and lab testing must continue to be done so that confidence in the use of these methods may continue to grow.

The use of probabilistic concepts in geophysical exploration for the prediction of site response to structural loadings cannot be overlooked as a means of enhancing engineering decision making. When its potential is on the way to being fully realized, the present methods of communicating degrees of uncertainty, of comparing risks, and of evaluating experience will be enhanced tremendously.

REFERENCES

1. Christian, J. T. (1979),"Probabilistic Soil Dynamics," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Reliability Analysis and Geotechnical Engineering, April 2-6. Boston, Massachusetts, pg 136-161.

2. Griffiths, D. H., and King, R. F. (1965). Applied Geophysics For Engineers and Geologists, Pergamon Press, Great Britain.

3. Harr, M. E. (1977). Mechanics of Particulate Media, McGraw-Hill Book Company, New York.

4. Marcuson, W. F., and Curro, J. R. Jr. (1979),"Field and Laboratory Determination of Soil Moduli," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Geophysical Methods in Geotechnical Engineering, October 23-25. Atlanta, Georgia, pg 243-278.

5. McEvilly, T. V., and Nelson, J. S. (1979), "Seismic Methods in Geotechnical Engineering," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Geophysical Methods in Geotechnical Engineering, October 23-25. Atlanta, Georgia, pg 91-112.

6. Nieto, A. S. (1978), "Probabilistic Methods and Site Characterization," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Site Characterization and Exploration, June 12-14. Evanston, Illinois, pg 252-257.

7. Romig, P. (1974), "New Applications of Seismic Investigations To Engineering Problems," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Subsurface Exploration For Underground Excavation and Heavy Construction, August 11-16. Henniker, New Hampshire, pg 144-158.

8. Shaw, D. E., and D'Appolonia, E. (1979), "Decision Analysis in Geotechnical Engineering," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Reliability Analysis and Geotechnical Engineering, April 2-6. Boston, Massachusetts, pg 119-135.

9. Viksne, A. (1979), "Exploration Geophysics For Site Evaluation of Engineering Works," Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, Geophysical Methods in Geotechnical Engineering, October 23-25. Atlanta, Georgia, pg 323-341.

10. Engineer Manual 1110-1-1802, Geophysical Explorations, Department of the Army, Corps of Engineers, U.S.A., 31 May 1979.

APPENDIX

Results of the Variability Study

for

V_p = 1000 m/sec
V_s = 200 m/sec
ρ = 200kg/cubic meter
V_e = 5 %

$$\frac{V_s}{V_p} = 0.2$$

```
INPUT PARAMETERS

COMPRESSIONAL WAVE VELOCITY,        VP =  1.000E+03
 COEFFICIENT OF VARIATION,  COEFVP =  5.000E+00
          STANDARD DEVIATION, SDVP =  5.000E+01

SHEAR WAVE VELOCITY,                VS =  2.000E+02
 COEFFICIENT OF VARIATION,  COEFVS =  5.000E+00
          STANDARD DEVIATION, SDVS =  1.000E+01

DENSITY,                             P =  2.000E+02
  COEFFICIENT OF VARIATION,  COEFP =  5.000E+00
           STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                     MEAN =  2.321E+02
                 VARIANCE =  6.735E+02
 COEFFICIENT OF VARIATION =  1.118E-01
       STANDARD DEVIATION =  2.595E+01

POISSONS RATIO, V

                     MEAN =  4.792E-01
                 VARIANCE =  9.419E-06
 COEFFICIENT OF VARIATION =  6.405E-03
       STANDARD DEVIATION =  3.069E-03

BULK MODULUS, B

                     MEAN =  1.857E+03
                 VARIANCE =  4.719E+04
 COEFFICIENT OF VARIATION =  1.170E-01
       STANDARD DEVIATION =  2.172E+02

SHEAR MODULUS, G

                     MEAN =  7.845E+01
                 VARIANCE =  7.693E+01
 COEFFICIENT OF VARIATION =  1.118E-01
       STANDARD DEVIATION =  8.771E+00
```

```
INPUT PARAMETERS

COMPRESSIONAL WAVE VELOCITY,        VP =  1.000E+03
 COEFFICIENT OF VARIATION,  COEFVP =  5.000E+00
          STANDARD DEVIATION, SDVP =  5.000E+01

SHEAR WAVE VELOCITY,                VS =  2.000E+02
 COEFFICIENT OF VARIATION,  COEFVS =  1.000E+01
          STANDARD DEVIATION, SDVS =  2.000E+01

DENSITY,                             P =  2.000E+02
  COEFFICIENT OF VARIATION,  COEFP =  5.000E+00
           STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                     MEAN =  2.321E+02
                 VARIANCE =  2.290E+03
 COEFFICIENT OF VARIATION =  2.062E-01
       STANDARD DEVIATION =  4.785E+01

POISSONS RATIO, V

                     MEAN =  4.792E-01
                 VARIANCE =  2.355E-05
 COEFFICIENT OF VARIATION =  1.013E-02
       STANDARD DEVIATION =  4.853E-03

BULK MODULUS, B

                     MEAN =  1.857E+03
                 VARIANCE =  4.752E+04
 COEFFICIENT OF VARIATION =  1.174E-01
       STANDARD DEVIATION =  2.180E+02

SHEAR MODULUS, G

                     MEAN =  7.845E+01
                 VARIANCE =  2.615E+02
 COEFFICIENT OF VARIATION =  2.062E-01
       STANDARD DEVIATION =  1.617E+01
```

```
INF  PARAMETERS

   COMPRESSIONAL WAVE VELOCITY,      VP =  1.000E+03
    COEFFICIENT OF VARIATION, COEFVP =  5.000E+00
            STANDARD DEVIATION, SDVP =  5.000E+01

   SHEAR WAVE VELOCITY,               VS =  2.000E+02
    COEFFICIENT OF VARIATION, COEFVS =  1.500E+01
            STANDARD DEVIATION, SDVS =  3.000E+01

   DENSITY,                            P =  2.000E+02
     COEFFICIENT OF VARIATION, COEFP =  5.000E+00
             STANDARD DEVIATION, SDP =  1.000E+01


  YOUNGS MODULUS, E

                         MEAN =  2.321E+02
                     VARIANCE =  4.983E+03
     COEFFICIENT OF VARIATION =  3.042E-01
           STANDARD DEVIATION =  7.059E+01

  POISSONS RATIO, V

                         MEAN =  4.792E-01
                     VARIANCE =  4.710E-05
     COEFFICIENT OF VARIATION =  1.432E-02
           STANDARD DEVIATION =  6.863E-03

  BULK MODULUS, B

                         MEAN =  1.857E+03
                     VARIANCE =  4.807E+04
     COEFFICIENT OF VARIATION =  1.181E-01
           STANDARD DEVIATION =  2.192E+02

  SHEAR MODULUS, G

                         MEAN =  7.845E+01
                     VARIANCE =  5.693E+02
     COEFFICIENT OF VARIATION =  3.041E-01
           STANDARD DEVIATION =  2.386E+01
```

```
INPUT PARAMETERS

   COMPRESSIONAL WAVE VELOCITY,      VP =  1.000E+03
    COEFFICIENT OF VARIATION, COEFVP =  5.000E+00
            STANDARD DEVIATION, SDVP =  5.000E+01

   SHEAR WAVE VELOCITY,               VS =  2.000E+02
    COEFFICIENT OF VARIATION, COEFVS =  2.000E+01
            STANDARD DEVIATION, SDVS =  4.000E+01

   DENSITY,                            P =  2.000E+02
     COEFFICIENT OF VARIATION, COEFP =  5.000E+00
             STANDARD DEVIATION, SDP =  1.000E+01


  YOUNGS MODULUS, E

                         MEAN =  2.321E+02
                     VARIANCE =  8.754E+03
     COEFFICIENT OF VARIATION =  4.032E-01
           STANDARD DEVIATION =  9.356E+01

  POISSONS RATIO, V

                         MEAN =  4.792E-01
                     VARIANCE =  8.006E-05
     COEFFICIENT OF VARIATION =  1.867E-02
           STANDARD DEVIATION =  8.948E-03

  BULK MODULUS, B

                         MEAN =  1.857E+03
                     VARIANCE =  4.883E+04
     COEFFICIENT OF VARIATION =  1.190E-01
           STANDARD DEVIATION =  2.210E+02

  SHEAR MODULUS, G

                         MEAN =  7.845E+01
                     VARIANCE =  1.000E+03
     COEFFICIENT OF VARIATION =  4.031E-01
           STANDARD DEVIATION =  3.162E+01
```

```
INPUT PARAMETERS

  COMPRESSIONAL WAVE VELOCITY,        VP =  1.000E+03
   COEFFICIENT OF VARIATION, COEFVP =  1.000E+01
           STANDARD DEVIATION, SDVP =  1.000E+02

  SHEAR WAVE VELOCITY,                VS =  2.000E+02
   COEFFICIENT OF VARIATION, COEFVS =  5.000E+00
           STANDARD DEVIATION, SDVS =  1.000E+01

  DENSITY,                             P =  2.000E+02
    COEFFICIENT OF VARIATION, COEFP =  5.000E+00
            STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                        MEAN =  2.321E+02
                    VARIANCE =  6.738E+02
    COEFFICIENT OF VARIATION =  1.119E-01
          STANDARD DEVIATION =  2.596E+01

POISSONS RATIO, V

                        MEAN =  4.792E-01
                    VARIANCE =  2.355E-05
    COEFFICIENT OF VARIATION =  1.013E-02
          STANDARD DEVIATION =  4.853E-03

BULK MODULUS, B

                        MEAN =  1.857E+03
                    VARIANCE =  1.626E+05
    COEFFICIENT OF VARIATION =  2.172E-01
          STANDARD DEVIATION =  4.032E+02

SHEAR MODULUS, G

                        MEAN =  7.845E+01
                    VARIANCE =  7.693E+01
    COEFFICIENT OF VARIATION =  1.118E-01
          STANDARD DEVIATION =  8.771E+00
```

```
INPUT PARAMETERS

  COMPRESSIONAL WAVE VELOCITY,        VP =  1.000E+03
   COEFFICIENT OF VARIATION, COEFVP =  1.000E+01
           STANDARD DEVIATION, SDVP =  1.000E+02

  SHEAR WAVE VELOCITY,                VS =  2.000E+02
   COEFFICIENT OF VARIATION, COEFVS =  1.000E+01
           STANDARD DEVIATION, SDVS =  2.000E+01

  DENSITY,                             P =  2.000E+02
    COEFFICIENT OF VARIATION, COEFP =  5.000E+00
            STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                        MEAN =  2.321E+02
                    VARIANCE =  2.290E+03
    COEFFICIENT OF VARIATION =  2.062E-01
          STANDARD DEVIATION =  4.785E+01

POISSONS RATIO, V

                        MEAN =  4.792E-01
                    VARIANCE =  3.768E-05
    COEFFICIENT OF VARIATION =  1.281E-02
          STANDARD DEVIATION =  6.138E-03

BULK MODULUS, B

                        MEAN =  1.857E+03
                    VARIANCE =  1.629E+05
    COEFFICIENT OF VARIATION =  2.174E-01
          STANDARD DEVIATION =  4.036E+02

SHEAR MODULUS, G

                        MEAN =  7.845E+01
                    VARIANCE =  2.615E+02
    COEFFICIENT OF VARIATION =  2.062E-01
          STANDARD DEVIATION =  1.617E+01
```

```
INPUT PARAMETERS

COMPRESSIONAL WAVE VELOCITY,         VP =  1.000E+03
 COEFFICIENT OF VARIATION, COEFVP =  1.000E+01
        STANDARD DEVIATION, SDVP =  1.000E+02

SHEAR WAVE VELOCITY,                 VS =  2.000E+02
 COEFFICIENT OF VARIATION, COEFVS =  1.500E+01
        STANDARD DEVIATION, SDVS =  3.000E+01

DENSITY,                              P =  2.000E+02
  COEFFICIENT OF VARIATION, COEFP =  5.000E+00
         STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                     MEAN =  2.321E+02
                 VARIANCE =  4.983E+03
 COEFFICIENT OF VARIATION =  3.042E-01
       STANDARD DEVIATION =  7.059E+01

POISSONS RATIO, V

                     MEAN =  4.792E-01
                 VARIANCE =  6.122E-05
 COEFFICIENT OF VARIATION =  1.633E-02
       STANDARD DEVIATION =  7.825E-03

BULK MODULUS, B

                     MEAN =  1.857E+03
                 VARIANCE =  1.635E+05
 COEFFICIENT OF VARIATION =  2.178E-01
       STANDARD DEVIATION =  4.043E+02

SHEAR MODULUS, G

                     MEAN =  7.845E+01
                 VARIANCE =  5.693E+02
 COEFFICIENT OF VARIATION =  3.041E-01
       STANDARD DEVIATION =  2.386E+01
```

```
INPUT PARAMETERS

COMPRESSIONAL WAVE VELOCITY,         VP =  1.000E+03
 COEFFICIENT OF VARIATION, COEFVP =  1.000E+01
        STANDARD DEVIATION, SDVP =  1.000E+02

SHEAR WAVE VELOCITY,                 VS =  2.000E+02
 COEFFICIENT OF VARIATION, COEFVS =  2.000E+01
        STANDARD DEVIATION, SDVS =  4.000E+01

DENSITY,                              P =  2.000E+02
  COEFFICIENT OF VARIATION, COEFP =  5.000E+00
         STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                     MEAN =  2.321E+02
                 VARIANCE =  8.754E+03
 COEFFICIENT OF VARIATION =  4.032E-01
       STANDARD DEVIATION =  9.356E+01

POISSONS RATIO, V

                     MEAN =  4.792E-01
                 VARIANCE =  9.419E-05
 COEFFICIENT OF VARIATION =  2.025E-02
       STANDARD DEVIATION =  9.705E-03

BULK MODULUS, B

                     MEAN =  1.857E+03
                 VARIANCE =  1.642E+05
 COEFFICIENT OF VARIATION =  2.183E-01
       STANDARD DEVIATION =  4.052E+02

SHEAR MODULUS, G

                     MEAN =  7.845E+01
                 VARIANCE =  1.000E+03
 COEFFICIENT OF VARIATION =  4.031E-01
       STANDARD DEVIATION =  3.162E+01
```

INPUT PARAMETERS

```
COMPRESSIONAL WAVE VELOCITY,          VP = 1.000E+03
 COEFFICIENT OF VARIATION, COEFVP =        1.500E+01
          STANDARD DEVIATION, SDVP =       1.500E+02

SHEAR WAVE VELOCITY,                  VS = 2.000E+02
 COEFFICIENT OF VARIATION, COEFVS =        5.000E+00
          STANDARD DEVIATION, SDVS =       1.000E+01

DENSITY,                               P = 2.000E+02
  COEFFICIENT OF VARIATION, COEFP =        5.000E+00
           STANDARD DEVIATION, SDP =       1.000E+01


YOUNGS MODULUS, E

                          MEAN =  2.321E+02
                      VARIANCE =  6.744E+02
      COEFFICIENT OF VARIATION =  1.119E-01
            STANDARD DEVIATION =  2.597E+01

POISSONS RATIO, V

                          MEAN =  4.792E-01
                      VARIANCE =  4.710E-05
      COEFFICIENT OF VARIATION =  1.432E-02
            STANDARD DEVIATION =  6.863E-03

BULK MODULUS, B

                          MEAN =  1.857E+03
                      VARIANCE =  3.549E+05
      COEFFICIENT OF VARIATION =  3.209E-01
            STANDARD DEVIATION =  5.957E+02

SHEAR MODULUS, G

                          MEAN =  7.845E+01
                      VARIANCE =  7.693E+01
      COEFFICIENT OF VARIATION =  1.118E-01
            STANDARD DEVIATION =  8.771E+00
```

INPUT PARAMETERS

```
COMPRESSIONAL WAVE VELOCITY,          VP = 1.000E+03
 COEFFICIENT OF VARIATION, COEFVP =        1.500E+01
          STANDARD DEVIATION, SDVP =       1.500E+02

SHEAR WAVE VELOCITY,                  VS = 2.000E+02
 COEFFICIENT OF VARIATION, COEFVS =        1.000E+01
          STANDARD DEVIATION, SDVS =       2.000E+01

DENSITY,                               P = 2.000E+02
  COEFFICIENT OF VARIATION, COEFP =        5.000E+00
           STANDARD DEVIATION, SDP =       1.000E+01


YOUNGS MODULUS, E

                          MEAN =  2.321E+02
                      VARIANCE =  2.291E+03
      COEFFICIENT OF VARIATION =  2.062E-01
            STANDARD DEVIATION =  4.786E+01

POISSONS RATIO, V

                          MEAN =  4.792E-01
                      VARIANCE =  6.122E-05
      COEFFICIENT OF VARIATION =  1.633E-02
            STANDARD DEVIATION =  7.825E-03

BULK MODULUS, B

                          MEAN =  1.857E+03
                      VARIANCE =  3.552E+05
      COEFFICIENT OF VARIATION =  3.210E-01
            STANDARD DEVIATION =  5.960E+02

SHEAR MODULUS, G

                          MEAN =  7.845E+01
                      VARIANCE =  2.615E+02
      COEFFICIENT OF VARIATION =  2.062E-01
            STANDARD DEVIATION =  1.617E+01
```

D5HC5CC 13.C8.05.LP311 13 PAGES

```
IN  f PARAMETERS

 COMPRESSIONAL WAVE VELOCITY,     VP =   1.000E+03
  COEFFICIENT OF VARIATION, COEFVP =   1.500E+01
           STANDARD DEVIATION, SDVP =   1.500E+02

 SHEAR WAVE VELOCITY,             VS =   2.000E+02
  COEFFICIENT OF VARIATION, COEFVS =   1.500E+01
           STANDARD DEVIATION, SDVS =   3.000E+01

 DENSITY,                          P =   2.900E+02
   COEFFICIENT OF VARIATION, COEFP =   5.000E+00
            STANDARD DEVIATION, SDP =   1.000E+01


YOUNGS MODULUS, E

                     MEAN =  2.321E+02
                 VARIANCE =  4.984E+03
 COEFFICIENT OF VARIATION =  3.042E-01
       STANDARD DEVIATION =  7.060E+01

POISSONS RATIO, V

                     MEAN =  4.792E-01
                 VARIANCE =  8.477E-05
 COEFFICIENT OF VARIATION =  1.921E-02
       STANDARD DEVIATION =  9.207E-03

BULK MODULUS, B

                     MEAN =  1.857E+03
                 VARIANCE =  3.558E+05
 COEFFICIENT OF VARIATION =  3.213E-01
       STANDARD DEVIATION =  5.965E+02

SHEAR MODULUS, G

                     MEAN =  7.845E+01
                 VARIANCE =  5.693E+02
 COEFFICIENT OF VARIATION =  3.041E-01
       STANDARD DEVIATION =  2.386E+01
```

```
INPUT PARAMETERS

 COMPRESSIONAL WAVE VELOCITY,     VP =   1.000E+03
  COEFFICIENT OF VARIATION, COEFVP =   1.500E+01
           STANDARD DEVIATION, SDVP =   1.500E+02

 SHEAR WAVE VELOCITY,             VS =   2.000E+02
  COEFFICIENT OF VARIATION, COEFVS =   2.000E+01
           STANDARD DEVIATION, SDVS =   4.000E+01

 DENSITY,                          P =   2.000E+02
   COEFFICIENT OF VARIATION, COEFP =   5.000E+00
            STANDARD DEVIATION, SDP =   1.000E+01


YOUNGS MODULUS, E

                     MEAN =  2.321E+02
                 VARIANCE =  8.755E+03
 COEFFICIENT OF VARIATION =  4.032E-01
       STANDARD DEVIATION =  9.357E+01

POISSONS RATIO, V

                     MEAN =  4.792E-01
                 VARIANCE =  1.177E-04
 COEFFICIENT OF VARIATION =  2.264E-02
       STANDARD DEVIATION =  1.085E-02

BULK MODULUS, B

                     MEAN =  1.857E+03
                 VARIANCE =  3.565E+05
 COEFFICIENT OF VARIATION =  3.216E-01
       STANDARD DEVIATION =  5.971E+02

SHEAR MODULUS, G

                     MEAN =  7.845E+01
                 VARIANCE =  1.000E+03
 COEFFICIENT OF VARIATION =  4.031E-01
       STANDARD DEVIATION =  3.162E+01
```

INP PARAMETERS

COMPRESSIONAL WAVE VELOCITY,	VP =	1.000E+03
COEFFICIENT OF VARIATION,	COEFVP =	2.000E+01
STANDARD DEVIATION,	SDVP =	2.000E+02
SHEAR WAVE VELOCITY,	VS =	2.000E+02
COEFFICIENT OF VARIATION,	COEFVS =	5.000E+00
STANDARD DEVIATION,	SDVS =	1.000E+01
DENSITY,	P =	2.000E+02
COEFFICIENT OF VARIATION,	COEFP =	5.000E+00
STANDARD DEVIATION,	SDP =	1.000E+01

YOUNGS MODULUS, E

MEAN =	2.321E+02
VARIANCE =	6.752E+02
COEFFICIENT OF VARIATION =	1.120E-01
STANDARD DEVIATION =	2.598E+01

POISSONS RATIO, V

MEAN =	4.792E-01
VARIANCE =	8.006E-05
COEFFICIENT OF VARIATION =	1.867E-02
STANDARD DEVIATION =	8.948E-03

BULK MODULUS, B

MEAN =	1.857E+03
VARIANCE =	6.241E+05
COEFFICIENT OF VARIATION =	4.255E-01
STANDARD DEVIATION =	7.900E+02

SHEAR MODULUS, G

MEAN =	7.845E+01
VARIANCE =	7.693E+01
COEFFICIENT OF VARIATION =	1.118E-01
STANDARD DEVIATION =	8.771E+00

INPUT PARAMETERS

COMPRESSIONAL WAVE VELOCITY,	VP =	1.000E+03
COEFFICIENT OF VARIATION,	COEFVP =	2.000E+01
STANDARD DEVIATION,	SDVP =	2.000E+02
SHEAR WAVE VELOCITY,	VS =	2.000E+02
COEFFICIENT OF VARIATION,	COEFVS =	1.000E+01
STANDARD DEVIATION,	SDVS =	2.000E+01
DENSITY,	P =	2.000E+02
COEFFICIENT OF VARIATION,	COEFP =	5.000E+00
STANDARD DEVIATION,	SDP =	1.000E+01

YOUNGS MODULUS, E

MEAN =	2.321E+02
VARIANCE =	2.291E+03
COEFFICIENT OF VARIATION =	2.063E-01
STANDARD DEVIATION =	4.787E+01

POISSONS RATIO, V

MEAN =	4.792E-01
VARIANCE =	9.419E-05
COEFFICIENT OF VARIATION =	2.025E-02
STANDARD DEVIATION =	9.705E-03

BULK MODULUS, B

MEAN =	1.857E+03
VARIANCE =	6.245E+05
COEFFICIENT OF VARIATION =	4.256E-01
STANDARD DEVIATION =	7.902E+02

SHEAR MODULUS, G

MEAN =	7.845E+01
VARIANCE =	2.615E+02
COEFFICIENT OF VARIATION =	2.062E-01
STANDARD DEVIATION =	1.617E+01

```
INP   PARAMETERS

  COMPRESSIONAL WAVE VELOCITY,     VP =  1.000E+03
   COEFFICIENT OF VARIATION, COEFVP =  2.000E+01
          STANDARD DEVIATION, SDVP =  2.000E+02

  SHEAR WAVE VELOCITY,              VS =  2.000E+02
   COEFFICIENT OF VARIATION, COEFVS =  1.500E+01
          STANDARD DEVIATION, SDVS =  3.000E+01

  DENSITY,                           P =  2.000E+02
    COEFFICIENT OF VARIATION, COEFP =  5.000E+00
           STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                           MEAN =  2.321E+02
                       VARIANCE =  4.985E+03
       COEFFICIENT OF VARIATION =  3.042E-01
             STANDARD DEVIATION =  7.060E+01

POISSONS RATIO, V

                           MEAN =  4.792E-01
                       VARIANCE =  1.177E-04
       COEFFICIENT OF VARIATION =  2.264E-02
             STANDARD DEVIATION =  1.085E-02

BULK MODULUS, B

                           MEAN =  1.857E+03
                       VARIANCE =  6.250E+05
       COEFFICIENT OF VARIATION =  4.258E-01
             STANDARD DEVIATION =  7.906E+02

SHEAR MODULUS, G

                           MEAN =  7.845E+01
                       VARIANCE =  5.693E+02
       COEFFICIENT OF VARIATION =  3.041E-01
             STANDARD DEVIATION =  2.386E+01
```

```
INPUT PARAMETERS

  COMPRESSIONAL WAVE VELOCITY,     VP =  1.000E+03
   COEFFICIENT OF VARIATION, COEFVP =  2.000E+01
          STANDARD DEVIATION, SDVP =  2.000E+02

  SHEAR WAVE VELOCITY,              VS =  2.000E+02
   COEFFICIENT OF VARIATION, COEFVS =  2.000E+01
          STANDARD DEVIATION, SDVS =  4.000E+01

  DENSITY,                           P =  2.000E+02
    COEFFICIENT OF VARIATION, COEFP =  5.000E+00
           STANDARD DEVIATION, SDP =  1.000E+01


YOUNGS MODULUS, E

                           MEAN =  2.321E+02
                       VARIANCE =  8.756E+03
       COEFFICIENT OF VARIATION =  4.032E-01
             STANDARD DEVIATION =  9.357E+01

POISSONS RATIO, V

                           MEAN =  4.792E-01
                       VARIANCE =  1.507E-04
       COEFFICIENT OF VARIATION =  2.562E-02
             STANDARD DEVIATION =  1.228E-02

BULK MODULUS, B

                           MEAN =  1.857E+03
                       VARIANCE =  6.258E+05
       COEFFICIENT OF VARIATION =  4.261E-01
             STANDARD DEVIATION =  7.911E+02

SHEAR MODULUS, G

                           MEAN =  7.845E+01
                       VARIANCE =  1.000E+03
       COEFFICIENT OF VARIATION =  4.031E-01
             STANDARD DEVIATION =  3.162E+01


D5HC50C  13.08.46.LP311    11 PAGES
```